

## 5.1 Aturan Trapezium

Integral tentu dari fungsi  $f(x)$  pada selang  $[a, b]$ , yaitu

$$I = \int_a^b f(x) dx \quad (5.1.1)$$

Menghitung integral (5.1.1) artinya menghitung luas daerah yang diarsir. Misalkan selang  $[a, b]$ , kita bagi menjadi  $N$  selang kecil-kecil yang lebarnya sama, dengan  $(a, f(a)) = (x_0, f_0)$ . Berdasarkan interpolasi linier menggunakan titik  $(x_0, f_0)$  dan titik  $(x_1, f_1)$  diperoleh rumus untuk fungsi  $f$  adalah

$$\frac{f - f_0}{f_1 - f_0} = \frac{x - x_0}{x_1 - x_0}$$

$$f = \frac{x - x_0}{x_1 - x_0} (f_1 - f_0) + f_0$$

Jika  $(x_0, f_0) = (a, f_0)$  dan titik  $(x_1, f_1)$ , maka

$$f = \frac{x - a}{x_1 - a} (f_1 - f_0) + f_0 = \frac{1}{x_1 - a} f_1 x - \frac{a}{x_1 - a} f_1 - \frac{1}{x_1 - a} f_0 x + \frac{x_1}{x_1 - a} f_0$$

$$\int_a^{x_1} f(x) dx = \int_a^{x_1} \left( \frac{1}{x_1 - a} f_1 x - \frac{a}{x_1 - a} f_1 - \frac{1}{x_1 - a} f_0 x + \frac{x_1}{x_1 - a} f_0 \right) dx$$

$$\int_a^{x_1} f(x) dx = \left[ \frac{1}{2(x_1 - a)} f_1 x^2 - \frac{a}{x_1 - a} f_1 x - \frac{1}{2(x_1 - a)} f_0 x^2 + \frac{x_1}{x_1 - a} f_0 x \right]_a^{x_1}$$

$$\begin{aligned} \int_a^{x_1} f(x) dx &= \left[ \frac{1}{2(x_1 - a)} f_1 x_1^2 - \frac{a}{x_1 - a} f_1 x_1 - \frac{1}{2(x_1 - a)} f_0 x_1^2 + \frac{x_1}{x_1 - a} f_0 x_1 \right] - \\ &\quad \left[ \frac{1}{2(x_1 - a)} f_1 a^2 - \frac{a}{x_1 - a} f_1 a - \frac{1}{2(x_1 - a)} f_0 a^2 + \frac{x_1}{x_1 - a} f_0 a \right] \end{aligned}$$

$$\int_a^{x_1} f(x) dx = \frac{(x_1^2 - a^2)}{2(x_1 - a)} f_1 - \frac{a(x_1 - a)}{x_1 - a} f_1 - \frac{(x_1^2 - a^2)}{2(x_1 - a)} f_0 + \frac{x_1(x_1 - a)}{x_1 - a} f_0$$

$$\int_a^{x_1} f(x)dx = \frac{(x_1 + a)}{2} f_1 - af_1 - \frac{(x_1 + a)}{2} f_0 + x_1 f_0$$

$$\int_a^{x_1} f(x)dx = \frac{x_1}{2} f_1 + \frac{a}{2} f_1 - af_1 - \frac{x_1}{2} f_0 - \frac{a}{2} f_0 + x_1 f_0$$

$$\int_a^{x_1} f(x)dx = \frac{x_1 - a}{2} f_1 + \frac{x_1 - a}{2} f_0 = \frac{x_1 - a}{2} f_0 + \frac{x_1 - a}{2} f_1 = \frac{x_1 - a}{2} (f_0 + f_1)$$

Dengan cara yang sama diperoleh

$$\int_{x_1}^{x_2} f(x)dx = \frac{x_2 - x_1}{2} (f_1 + f_2)$$

$$\int_{x_2}^{x_3} f(x)dx = \frac{x_3 - x_2}{2} (f_2 + f_3)$$

⋮

$$\int_{x_{N-1}}^{x_b} f(x)dx = \frac{x_b - x_{N-1}}{2} (f_{N-1} + f_b)$$

$$I = \int_a^b f(x)dx = \int_a^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \cdots + \int_{x_{N-1}}^b f(x)dx$$

$$I = \int_a^b f(x)dx = \frac{x_1 - a}{2} (f_0 + f_1) + \frac{x_2 - x_1}{2} (f_1 + f_2) + \frac{x_3 - x_2}{2} (f_2 + f_3) + \cdots + \frac{x_b - x_{N-1}}{2} (f_{N-1} + f_b) \text{ Jika}$$

$$x_1 - a = x_2 - x_1 = x_3 - x_2 = \cdots = x_b - x_{N-1} = h$$

Maka

$$I = \int_a^b f(x)dx = \frac{h}{2} ((f_0 + f_1) + (f_1 + f_2) + (f_2 + f_3) + \cdots + (f_{N-1} + f_b))$$

$$I = \int_a^b f(x)dx = \frac{h}{2} (f_0 + 2f_1 + 2f_2 + 2f_3 + \cdots + 2f_{N-1} + f_b) \quad (5.1.2)$$

Dengan

$N$  : banyaknya interval

$$h : \text{ukuran interval}, \quad h = \frac{b - a}{N}$$

$$f_0 = f(a), f_1 = f(a+h), f_i = f(a + ih), \dots, f_N = f(b),$$